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Numerical methods

MME308

Assignment 4

Grop.

Problem no: 3,7,11

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Problem 3

X	6	7	8	9	10	11
Y	**0.7782**	**0.8451**	**0.9031**	**0.9542**	**1.0000**	**1.0414**

Solution :

b)

Selective Tow Point Adjacent The Point You Need Fined

	X_0	X_1
X	**8**	**9**
Y	**0.9031**	**0.9542**

$$F_1(x) = f(x_0) + f[x_1, x_0](x - x_0)$$

$$f(x_0) = 0.9031$$

$$f[x_1,x_0] = \frac{f(x0)-f(x1)}{x0-x1} = \frac{0.9542-0.9031}{9-8}$$

$$f[x_0,x_1] = 0.0511$$

$$f_1(x) = 0.9031 + 0.051(x - 8)$$

$$f_1(x) = 0.4951 + 0.051 X$$

$$f_1(8.3) = 0.4951 + 0.051*(8.3)$$

$$=0.9192$$

The Absolute Relative Approximate Error

$$|\in_a| = \left|\frac{0.9191-0.9192}{0.9191}\right| \times 100$$

$$=0.01\%$$

b)

	X_0	X_1	X_2
X	**7**	**8**	**9**
Y	**0.8451**	**0.9031**	**0.9542**

$$F_2(x) = f(x_0) + f[x_1, x_0](x - x_0) + f[x_2, x_1, x_0](x - x_0)(x - x_1).$$

$$f[x_1,x_0] = 0.0511$$

$$f[x2,x1,x0]=\frac{f[x2,x1]-f[x1-x0]}{x2-x0} = \frac{\frac{f(x2)-f(x1)}{x2-x1}-\frac{f(x1)-f(x0)}{x1-x0}}{x2-x0} = \frac{\frac{0.9542-0.9031}{9-8}-\frac{0.9031-0.8451}{8-7}}{9-7}$$

$$f[x_2,x_1,x_0] = -0.0035$$

$$F_2(8.3) = 0.8451 + 0.0511(x - x_0) - 0.0035*(x - x_0)(x - x_1).$$

$$F_2(x) = 0.8451 + 0.0511(x - 7) - 0.0035(x - 7)(x - 8)$$

$$f_2(8.3) = 0.8451 + 0.0511*(8.3-7) - 0.0035(8.3-7)(8.3-8)$$

$$=0.9102$$

The Absolute Relative Approximate Error

$$|\in_a| = \left|\frac{0.9191-0.9102}{0.9191}\right| \times 100$$

=0.97%

__Problem 7__

The following data defines the sea-level concentration of dissolved oxygen for fresh water as function of temperature

T (c^0)	O (mg/L)
0	14.621
8	11.843
16	9.870
24	8.418
32	7.305
40	6.413

Estimate O(29) using (a) quadratic Lagrange polynomial .

(b) cubic Lagrange polynomial .

<u>*Solution*</u>

a) For quadratic interpolation, the concentration is given by

$$O(T) = \sum_{i=0}^{2} L_i(T)O(T_i)$$

$$= L_0(T)O(T_0) + L_1(T)O(T_1) + L_2(T)O(T_2)$$

Since we want to find the concentration at $T = 21$, and we are using a second order polynomial, we need to choose the three data points that are closest to $T = 21$ that also bracket $T = 21$ to evaluate it. The three points are $T_0 = 24,\ T_1 = 32,$ and $T_2 = 40$.

Then

$$T_0 = 24,\ O(T_0) = 8.418$$

$$T_1 = 32,\ O(T_1) = 7.305$$

$$T_2 = 40,\ O(T_2) = 6.413$$

gives

$$L_0(T) = \prod_{\substack{j=0 \\ j\neq 0}}^{2} \frac{T - T_j}{T_0 - T_j}$$

$$= \left(\frac{T - T_1}{T_0 - T_1}\right)\left(\frac{T - T_2}{T_0 - T_2}\right)$$

$$L_1(T) = \prod_{\substack{j=0 \\ j\neq 1}}^{2} \frac{T - T_j}{T_1 - T_j}$$

$$= \left(\frac{T - T_0}{T_1 - T_0}\right)\left(\frac{T - T_2}{T_1 - T_2}\right)$$

$$L_2(T) = \prod_{\substack{j=0 \\ j\neq 2}}^{2} \frac{T - T_j}{T_2 - T_j}$$

$$= \left(\frac{t - t_0}{t_2 - t_0}\right)\left(\frac{t - t_1}{t_2 - t_1}\right)$$

Hence

$$O(T) = \left(\frac{T - T_1}{T_0 - T_1}\right)\left(\frac{T - T_2}{T_0 - T_2}\right)O(T_0) + \left(\frac{T - T_0}{T\ - T_0}\right)\left(\frac{T - T_2}{T_1 - T_2}\right)O(T_1) + \left(\frac{T - T_0}{T_2 - T_0}\right)\left(\frac{T - T_1}{T_2 - T_1}\right)O(T_2),\ T_0 \le T \le T_2$$

$$T(16)=\frac{(29-32)(29-40)}{(24-32)(24-40)}(8.418)+\frac{(29-24)(29-40)}{(32-24)(32-40)}(7.305)$$
$$+\frac{(29-24)(29-32)}{(40-24)(40-32)}(6.413)$$

$$=(0.2578)(8.418)+(0.8594)(7.305)+(-0.1172)(6.413)$$
$$=7.694\quad C$$

Problem 7 b :

T (c^0)	O (mg/L)
0	14.621
8	11.843
16	9.870
24	8.418
32	7.305
40	6.413

Solution **:**

a) $$O(T)=\sum_{i=0}^{3}L_i(T)O(T_i)$$
$$=L_0(T)O(T_0)+L_1(T)O(T_1)+L_2(T)O(T_2)+L_3(T)O(T_3)$$

Then

$$T_0=16,\ O(T_0)=9.870$$
$$T_1=24,\ O(T_1)=8.418$$
$$T_2=32,\ O(T_2)=7.305$$
$$T_3=40,\ O(T_3)=6.413$$

gives

$$L_0(T)=\prod_{\substack{j=0\\ j\neq 0}}^{3}\frac{T-T_j}{T_0-T_j}$$
$$=\left(\frac{T-T_1}{T_0-T}\right)\left(\frac{T-T_2}{T_0-T_2}\right)\left(\frac{T-T_3}{T_0-T_3}\right)$$
$$L_1(T)=\prod_{\substack{j=0\\ j\neq 1}}^{3}\frac{T-T_j}{T_1-T_j}$$

$$= \left(\frac{T-T_0}{T_1-T_0}\right)\left(\frac{T-T_2}{T_1-T_2}\right)\left(\frac{T-T_3}{T_1-T_3}\right)$$

$$L_2(T) = \prod_{\substack{j=0\\ j\neq 2}}^{3} \frac{T-T_j}{T_2-T_j}$$

$$= \left(\frac{T-T_0}{T_2-T_0}\right)\left(\frac{T-T_1}{T_2-T_1}\right)\left(\frac{T-T_3}{T_2-T_3}\right)$$

$$L_3(T) = \prod_{\substack{j=0\\ j\neq 3}}^{3} \frac{T-T_j}{T_3-T_j}$$

$$= \left(\frac{T-T_0}{T_3-T_0}\right)\left(\frac{T-T_1}{T_3-T_1}\right)\left(\frac{T-T_2}{T_3-T_2}\right)$$

Hence

$$O(T) = \left(\frac{T-T_1}{T_0-T_1}\right)\left(\frac{T-T_2}{T_0-T_2}\right)\left(\frac{T-T_3}{T_0-T_3}\right)O(T_0) + \left(\frac{T-T_0}{T_1-T_0}\right)\left(\frac{T-T_2}{T_1-T_2}\right)\left(\frac{T-T_3}{T_1-T_3}\right)O(T_1)$$

$$+ \left(\frac{T-T_0}{T_2-T_0}\right)\left(\frac{T-T_1}{T_2-T_1}\right)\left(\frac{T-T_3}{T_2-T_3}\right)O(T_2) + \left(\frac{T-T_0}{T_3-T_0}\right)\left(\frac{T-T_1}{T_3-T_1}\right)\left(\frac{T-T_2}{T_3-T_2}\right)O(T_3),\ T_0 \le T \le T_3$$

$$O(29) = \frac{(29-24)(29-32)(29-40)}{(16-24)(16-32)(16-40)}(9.870) + \frac{(29-16)(29-32)(29-40)}{(24-16)(24-32)(24-40)}(8.418)$$

$$+ \frac{(29-16)(29-24)(29-40)}{(32-16)(32-24)(32-40)}(7.305)$$

$$+ \frac{(29-16)(29-24)(29-32)}{(40-16)(40-24)(40-32)}(6.314)$$

$$= (-0.0537)(9.870) + (0.419)(8.418) + (0.698)(7.305) + (-0.0635)(6.314)$$

$$= 7.695 \text{ C}$$

b)

$$|\epsilon_a| = \left|\frac{7.695-7.694}{7.695}\right| \times 100$$

$$= 0.01299\%$$

Problem 11

X	-2	1	-4	4	3	-1
y	-1	2	-53	59	24	4

Solution :

Coordinates	0st order	1-th order	2-th order
x_0	$f[x_0]$		
x_1	$f[x_1]$	$f[x_0, x_1]$	
x_2	$f[x_2]$	$f[x_1, x_2]$	$f[x_0, x_1, x_2]$
x_3	$f[x_3]$	$f[x_2, x_3]$	$f[x_1, x_2, x_3]$

x	Y	1^{th} order	2^{th} order	3^{th} order	4^{th} order	5^{th} order
-2	-1					
1	2	1				
-4	-53	11	-5			
4	59	14	1	1		
3	24	35	3	1	0	
-1	4	5	6	1	0	0

From the table is determine the degree of polynomial is **3**

X	Y	1^{th} order
-2	-1	
1	2	1

$F_1(x) = b_o + b_1(x-x_0)$

$F_1(0) = -1 + (0 + 2)$

$F_1(0) = 1$

From the table at x=0 , y=1